Crystalline topological phases and crosscap states in conformal field theories

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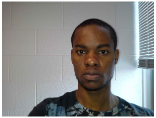
People and papers

Chang-Tse Hsieh (UIUC)



Bode Sule (UIUC)

Xiao Chen (UIUC)

Rob Leigh (UIUC)

Gil Young Cho (UIUC)

Ching-Kai Chiu (UIUC -> UBC)

Takahiro Morimoto (RIKEN)

Hong Yao (Tsinghua)

Shou-Cheng Zhang (Stanford)

SR and Shou-Cheng Zhang, PRB (2012).
H. Yao and SR, PRB (2013)
C.K. Chiu, H. Yao, SR, PRB (2013)
O. M. Sule, X. Chen, and SR, PRB (2013)
C.-T. Hsieh, T. Morimoto, SR, PRB (2014)
C.-T. Hsieh, O. M. Sule, G. Y. Cho, SR and R. G. Leigh, PRB (2014)

- Introduction

- Warmup: IQHE and QSHE

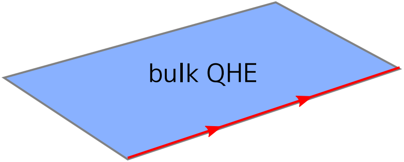
- Non-on-site symmetry, Crystalline topological insulators/SC

- Crystalline topological SC and interaction effects

"Sick" theories

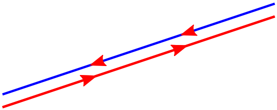
- Chiral edge of QHE

$$\mathcal{H} = \psi^\dagger i(v\partial_x)\psi$$



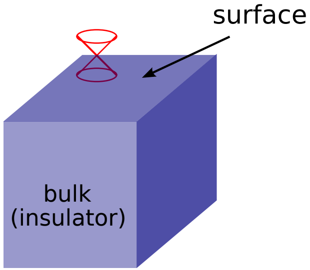
- Helical edge of QSHE

$$\mathcal{H} = \psi^\dagger i(v\partial_x\sigma_z)\psi \qquad \mathcal{T} = i\sigma_y K$$



- Surface of 3d topological insulators

$$\mathcal{H} = \psi^\dagger v\sigma\cdot k\psi \qquad \mathcal{T} = i\sigma_y K$$



What's "sick" about them?

- (Partial) answers:

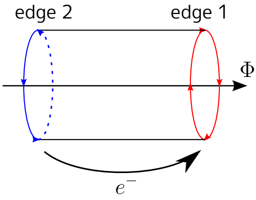
  They cannot be gapped (while preserving symmetries)! "Ingappable"
  They completely evade Anderson localization!
  ...

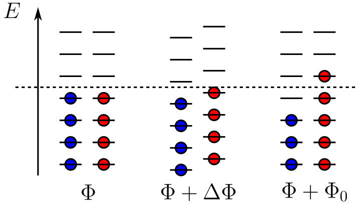
  --> Indicative of the absence of "atomic limit"

  --> These theories cannot be put on a lattice! (no-go theorem)

  --> Only possible chance: they live on a boundary
  of a higher-dimensional topological system

Laughlin's gauge argument





- Adiabatic process $\Phi \to \Phi + \Delta\Phi$

- When $\Delta\Phi = \text{integer} \times \Phi_0$ system goes back to itself
  ("large gauge equivalent") $H(\Phi) \equiv H(\Phi + n\Phi_0)$

- However, by this adiabatic process, an integer multiple of charge is transported from the left (right) to right (left) edge.

- Charge is not conserved for a given edge.

Topological phases and anomalies

Topological phases (in broad sense):
  no analogous phase in classical systems
  (very quantum state of matter)

Anomalies:
  breakdown of a classical symmetry by quantum effects
  (nothing is more quantum than this)

- A close relation known as bulk-boundary correspondence

- Advantage:
  - Robust against interactions, e.g., Adler-Bardeen's theorem
  - Observable: anomaly = "response"
    Operational definition of topological phases

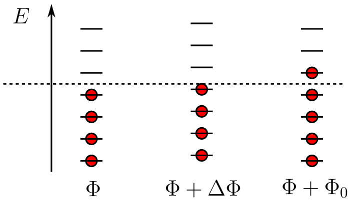
Laughlin's argument revisted

- Chiral edge theory

$$\mathcal{L} = \frac{1}{2\pi}\psi_R^\dagger i(\partial_t + v\partial_x)\psi_R$$

- Twisted boundary condition

$$\psi(x) - e^{2\pi i a}\psi(x+\ell) = 0$$



- Ground state with twisted bc ("twisted sector GS"): $|GS\rangle_a$

$$\left[\psi(x) - e^{2\pi i a}\psi(x+\ell)\right]|GS\rangle_a = 0$$

- "State-operator correspondence" $|GS\rangle_a = \mathcal{A}_a|GS\rangle_0$

$\mathcal{A}_a$: "twist operator"

$$\mathcal{A}_a \sim e^{-i(a-1/2)\varphi}$$

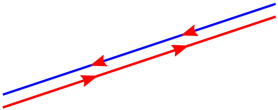
- The GS fermion number :

$$e^{ibQ}\mathcal{A}_a e^{-ibQ} = e^{ib(a-1/2)}\mathcal{A}_a$$

"QSHE" with conserved Sz

- Non-chiral edge theory

$$H = \int dx \left[\psi_\uparrow^\dagger(-i\partial_x)\psi_\uparrow + \psi_\downarrow^\dagger(i\partial_x)\psi_\downarrow \right]$$



- Twisted boundary condition by charge:

$$\psi_\sigma(x) - e^{2\pi i a}\psi_\sigma(x+\ell) = 0$$

- Twisted sector GS: $|GS\rangle_a = \mathcal{A}_a |GS\rangle_0$

$$\left[\psi_\sigma(x) - e^{2\pi i a}\psi_\sigma(x+\ell)\right] |GS_a\rangle = 0$$

- Twisted BC is invariant under Sz:

$$e^{ibS_z} \left[\psi_\sigma(x) - e^{2\pi i a}\psi_\sigma(x+\ell)\right] e^{-ibS_z} = 0$$

- The GS is not:

$$e^{ibS_z}|GS_a\rangle = e^{2iab}|GS_a\rangle$$

Lessons:

- Symmetries in QFTs can be twisted.

- Once twisted, symmetry is "bulit-in" -- it is a part of the theory.

  Symmetry --> twist operator

  (convenient in studying SPTs)

- Twist operator or twisted sector GS may show anomalous behavior

  E.g. Twisted theory may fail to be modular invariant

  E.g. Twist operators may show fractional statistics

SPT phases protected by spatial symmetries

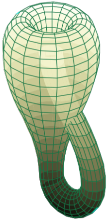
- Onsite v.s. non-onsite symmetries.

  This talk: non-onsite symmetries

  E.g., Parity symmetry.
  Topological crystalline insulators
  Topological crystalline superconductor

- Is there an anomaly characterizing crystalline topological insulators and superconductors ?

- Proposed scheme --> "Orientifold" field theory
  (Edge) theories defined on non-orientalble space-time



Periodic table with reflection symmetry

Reflection	Class	C_q or R_q	$d=0$	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$
R	A	C_1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
R^+	AIII	C_0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
R^-	AIII	C_1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
R^+, R^{++}	AI	R_1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
	BDI	R_2	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
	D	R_3	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
	DIII	R_4	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
	AII	R_5	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CII	R_6	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
	C	R_7	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	R_0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
R^-, R^{--}	AI	R_7	0	0	0	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$
	BDI	R_0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$
	D	R_1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"
	DIII	R_2	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
	AII	R_3	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
	CII	R_4	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
	C	R_5	0	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
	CI	R_6	0	0	$\mathbb{Z}$	0	"$\mathbb{Z}_2$"	$\mathbb{Z}_2$	$\mathbb{Z}$	0
R^{+-}	BDI	R_1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
R^{-+}	DIII	R_3	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
R^{+-}	CII	R_5	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
R^{-+}	CI	R_7	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
R^{-+}	BDI, CII	C_1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
R^{+-}	DIII, CI	C_1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

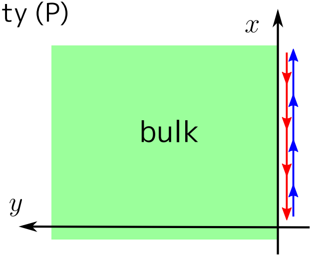
Chiu-Yao-SR (2013)
Morimoto-Furusaki (2013)
Shiozaki-Sato (2014)
...

- Systematic classification with reflection symmetry for non-interacting cases

Topological crystalline superconductors

- Topological superconductor protected by parity (P)

$$P : (x, y) \to (-x, y)$$



- Edge BdG Hamiltonian:

$$H = \int dx \, [\psi_L(-i\partial_x)\psi_L + \psi_R(i\partial_x)\psi_R]$$

- P symmetry $$P\psi_R(x)P^{-1} = \psi_L(-x)$$
$$P\psi_L(x)P^{-1} = \psi_R(-x)$$

- Can check no mass terms are allowed. Classification: Z2

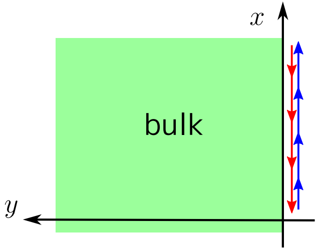
- With additional TRS, classification is Z $$T\psi_L(x)T^{-1} = \psi_R(x)$$
$$T\psi_L(x)T^{-1} = -\psi_R(x)$$

- How about interactions ? Z --> Z8

CP symmetric topological insulator

- System with CP and charge U(1) symmetries

$$P : (x, y) \to (-x, y)$$



"CPT-dual" of QSHE

- Edge Hamiltonian:

$$H = \int dx \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R\right]$$

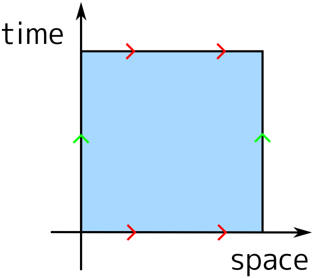
- CP symmetry

$$\begin{aligned} \mathcal{U}\,\psi_L(x)\,\mathcal{U}^{-1} &= \psi_R^\dagger(-x), \\ \mathcal{U}\,\psi_R(x)\,\mathcal{U}^{-1} &= e^{2\pi i\epsilon}\psi_L^\dagger(-x). \end{aligned}$$

$$e^{2\pi i\epsilon} = \begin{cases} +1 & \text{topological} \\ -1 & \text{trivial} \end{cases}$$

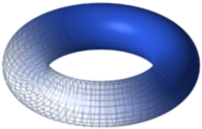
- Can check no mass terms are allowed when topological.
- How about interactions ?

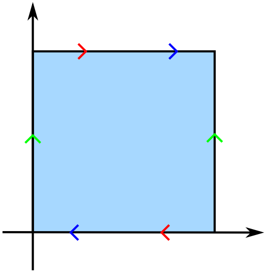
Twisting boundary conditions



Twisting by on-site symmetry

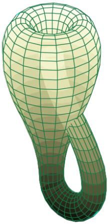
$$Z = \mathrm{Tr}_h \left[g e^{-\beta H} \right]$$
$$\Phi(t+T, x) = g \cdot \Phi(t, x)$$
$$\Phi(t, x+L) = h \cdot \Phi(t, x)$$





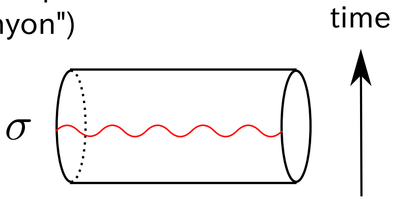
Twisting by parity symmetry

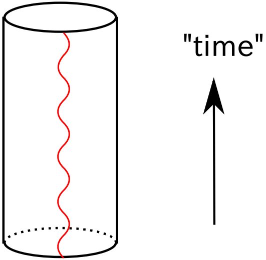
$$Z = \mathrm{Tr}_h \left[P\, e^{-\beta H} \right]$$
$$\Phi(t+T, x) = g \cdot \Phi(t, L-x)$$
$$\Phi(t, x+L) = h \cdot \Phi(t, x)$$



Twisting symmetry -- general strategy

twist operator
("anyon")





$|\sigma\rangle$

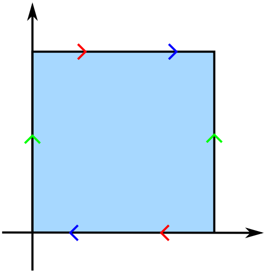
Twisted B.C.

"Twist state"

$$\Phi(z,\bar{z})\sigma(w,\bar{w}) = \sigma(w,\bar{w})U_\sigma \cdot \Phi(z,\bar{z})$$

$$[\Phi(x+\beta) - U_\sigma \cdot \Phi(x)]\,|\sigma\rangle = 0$$

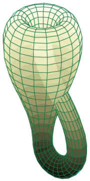
Crosscap and crosscap state



- Twisting b.c. by parity:

$$\Phi(x_1, x_2 + \beta) \quad = \quad \mathcal{P}\Phi(x_1, x_2)\mathcal{P}^{-1} = U \cdot \Phi(\ell - x_1, x_2)$$

- Klein bottle = sphere with two crosscap



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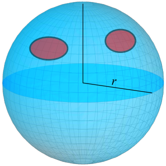
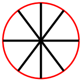
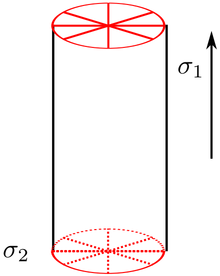




- Finding a nice time slice --> "Crosscap" state |C>:

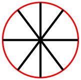


$$\left[\Phi(0, \sigma_2 + \beta) - U \cdot \Phi(0, \sigma_2)\right] |C\rangle = 0$$

Anomalous crosscap states

- Crosscap condition:

$$[\Phi(0, \sigma_2 + \beta) - U \cdot \Phi(0, \sigma_2)]\,|C\rangle = 0$$



circumference: 2β

c.f. Twisted sector ground state

$$[\Phi(x + \ell, \tau) - U_G \cdot \Phi(x, \tau)]\,|\sigma\rangle = 0$$

- Symmetry G acting on crosscap [e.g. G= U(1)]

$$\mathcal{G}\,[\Phi(0, \sigma_2 + \beta) - U \cdot \Phi(0, \sigma_2)]\,\mathcal{G}^{-1}\mathcal{G}|C\rangle = 0$$
$$\Rightarrow \quad \left[\Phi(0, \sigma_2 + \beta) - U_G^{-1} U U_G \Phi(0, \sigma_2)\mathcal{G}|C\rangle = 0\right]$$

- When U and UG commute, crosscap condition is invariant but crosscap state may not be!

$$\mathcal{G}|C\rangle = e^{i\alpha}|C\rangle$$

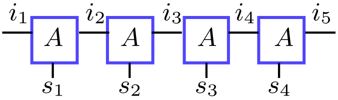
- Related to the anomalous phase of the partition function

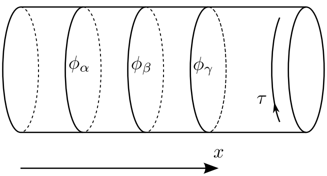
MPS (matrix product state) :

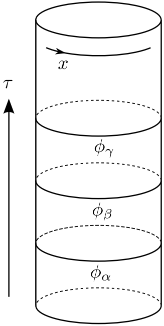
physical degrees of freedom

$$|\Psi\rangle = \sum_{\{s_a\}} \sum_{\{i_n=1,\cdots,\chi\}} A^{s_1}_{i_1,i_2} A^{s_2}_{i_2,i_3} A^{s_3}_{i_3,i_4} A^{s_4}_{i_4,i_5} \cdots |s_1, s_2, s_3, s_4 \ldots\rangle$$

auxiliary index







Analysis and result

- Symmetry group: $\{P, G_f, P \times G_f\}$

- Crosscap condition:

$$\left[\psi_L(0, \sigma^2 + \beta) + i\psi_R(0, \sigma^2)\right] |C, \eta\rangle = 0$$ twist by P

$$\left[\psi_L(0, \sigma^2 + \beta) - i\psi_R(0, \sigma^2)\right] |C, \eta\rangle = 0$$ twist by P x Gf

- Symmetry action on fermion number parity:

$$G_f |C, +\rangle = |C, +\rangle$$

$$G_f |C, -\rangle = -|C, -\rangle$$ "anomalous" relative phase

- Anomalous relative sign goes away for 2N copies --> Z2

- With time reversal: anomalous pi/2 relative phase --> Z4 ??

$$T|C, +\rangle = e^{i\phi_+}|C, +\rangle$$

$$T|C, -\rangle = e^{i\phi_-}|C, -\rangle$$

Summary

- Formulated Laughlin's argument for SPTs protected by parity and other symmetries

- Topology change from Torus to Klein

- Symmetry properties of crosscap states

- Reproduced expected Z_2 classification in all known cases

- Z_4 v.s. Z_8 ?